

Getting Ready For A Level Further Maths

Task 0.2 – Introduction to Complex Numbers

The square root of a negative number

Until now you may have learnt that you cannot square root a negative number, but now you are going to see how this is possible in the following way:

Suppose $i^2 = -1$ then we can deduce that $\sqrt{-1} = i$

This means we can use i every time we want to find the square root of a negative number.

Example: $\sqrt{-9} = \sqrt{9}\sqrt{-1} = 3i$ Check this by working backwards: $(3i)^2 = 9i^2 = 9 \times -1 = -9$

Powers of i

We can find other powers of i also:

Examples: $i^3 = i^2 \times i = -1 \times i = -i$ and $i^4 = i^2 \times i^2 = -1 \times -1 = 1$ etc....

Calculations with Complex Numbers

Complex numbers are any numbers that have i in them, such as $3 + 4i$ or $2 - 7i$.
You can calculate with them as long as you remember that $i^2 = -1$.

Examples: $(3 + 4i) + (5 - 7i) = 8 - 3i$

$(2 - 3i)(3 + 5i) = 6 - 9i + 10i - 15i^2 = 6 + i - (15 \times -1) = 6 + i + 15 = 21 + i$

***Remember to watch the videos for fuller explanations and demonstrations.**

Exercise

Now try these examples, simplifying as far as possible:

1) a) $\sqrt{-25}$ b) $\sqrt{-49}$ c) $\sqrt{-121}$

2) a) i^3 b) i^4 c) i^5 d) i^8 e) i^{34} f) i^{-1}

3) a) $(4 - 7i) + (2 - 6i)$ b) $(3 + 8i) - (1 + 5i)$ c) $(11 + i) - (12 - 2i)$

4) a) $(2 + 5i)(3 + 2i)$ b) $(3 + 7i)(4 - i)$ c) $(6 - 4i)(7 - 8i)$

5) Use the quadratic formula to solve for x and give answers as simplified as possible:

a) $x^2 + 4x + 5 = 0$

b) $x^2 + 6x + 11 = 0$

c) $2x^2 + 3x + 2 = 0$

6) Try these divisions, making the denominator real in the same way you would rationalise the denominator using surds (remember that $i = \sqrt{-1}$, so it is just like working with surds!)

a) $\frac{3+4i}{2+i}$

b) $\frac{22-2i}{4-5i}$

c) $\frac{2+4i}{5-3i}$

Answers

1) a) $5i$ b) $7i$ c) $11i$

2) a) $-i$ b) 1 c) i d) 1 e) -1 f) $-i$

3) a) $6 - 13i$ b) $2 + 3i$ c) $-1 + 3i$

4) a) $-4 + 19i$ b) $19 + 25i$ c) $10 - 76i$

5) a) $-2 \pm i$ b) $-3 \pm \sqrt{2}i$ c) $\frac{-3}{4} \pm \frac{\sqrt{7}}{4}i$

6) a) $2 + i$ b) $\frac{98}{41} + \frac{102}{41}i$ c) $\frac{-1}{17} + \frac{13}{17}i$