Chichester College

GCSE to A LEVEL

Summer Project

Name:
Subject: A Level Mathematics

The purpose of this A Level Summer project is to introduce you to studying this subject at A Level standard. You will need to complete 10 hours of study on each subject every week, 4½ in class with your teacher and the rest as independent learning. Therefore, it is important that you enjoy this subject and that you start to practice your study skills as early as possible. Some subjects have significant maths content (for example business, psychology, economics); others require strong essay writing skills (for example history, English). Think about the study skills and underpinning knowledge you will require in this subject – not just the title.

If after completing this project you think this may not be your ideal choice, you can ask to transfer to another subject at the start of term, as long as you have the entry requirements and it fits alongside your other choices on the A Level Matrix (timetable). If you do decide to change subject, you will be required to complete the Summer project for your new choice too.

This is also your first taste of Flipped Learning and elements will be used within your first week of lessons.

Please ensure your name, student number and subject are clearly noted on each page and bring it with you to hand in at Enrolment.

We hope you enjoy this project as you start your A Level journey.

Have a good summer and we look forward to seeing you in September.

HOW TO SUBMIT:

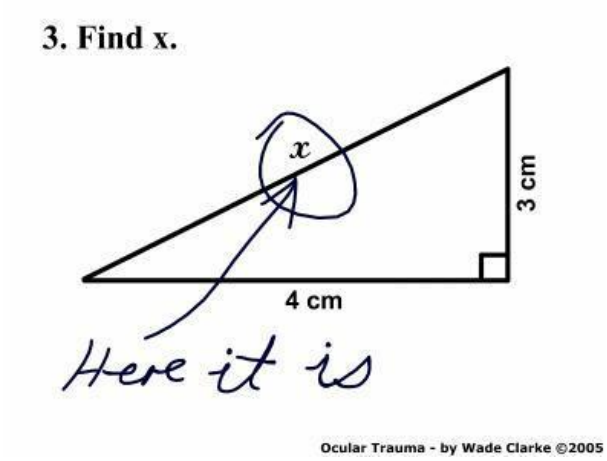
Please print your completed project and bring a copy with you to Induction.

If you don't have access to a printer, electronic copies can be emailed as an attachment to ALevel_Maths@chichester.ac.uk with the email clearly labelled 'Maths Summer Project' prior to Induction.

STARTING WITH CONFIDENCE

The last 3 pages of this booklet are very important! Don't forget to complete them!

This booklet has been designed to help you to bridge the gap between GCSE Maths and A Level Maths.



You will not be able to take A Level Maths at Chichester College unless you have completed this booklet to a good standard and brought it with you to enrolment after your GCSE results. On **1st week** you will be tested on some of these topics to ensure you have the skills needed to be successful.

When (not if) you get stuck DURING MATHS AT CHI…..

Studying Maths at advanced level is about learning how to solve problems. Solving a problem begins with us not knowing how to do it, so you should expect to get stuck while working through this booklet. Some of these topics may seem unfamiliar to you but they are all GCSE level topics and you need to be able to do all these techniques **before** you start AS Maths.

So, when you get stuck…

- Look at the recommended video clips.
- Look again at the examples. Maybe there is one which shows you how to solve your problem?
- Post a question on the Facebook wall 'Maths Life of Chi'
- Have you made a mistake? It might be that your method is correct but you've made an error in your working somewhere.
- Try looking up the topic in a GCSE higher tier textbook or revision guide (you can get these from your local library) or look online
- Call a friend who is also starting A Levels is September - you might be able to work it out between you

- If all else fails, message Maths team via facebook page "Chi Maths"; it may take a little while, but we will get back to you. Include your mobile number

CONTENTS

Part A – Learning to Avoid common algebraic 'Mistakes'.

We all make occasional mistakes when manipulating algebra and learning to make fewer mistakes (and finding the ones you have made!) is an important part of the study of maths at advanced level. However, there are also mistakes that aren't mistakes at all but are actually the result of a deeply held misunderstanding about the laws of algebra. These misunderstandings need to be exterminated as soon as possible. Do you understand why these examples are wrong? **Add them to your table of common mistakes on the back pages and think about how you know they are wrong.**

$$\frac{2+x^2}{3x} = \frac{2+x}{3}$$

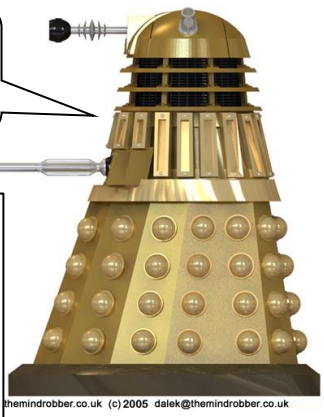
WRONG!! Exterminate!!

$$(a+b)^2 = a^2 + b^2$$

WRONG!! Exterminate!!

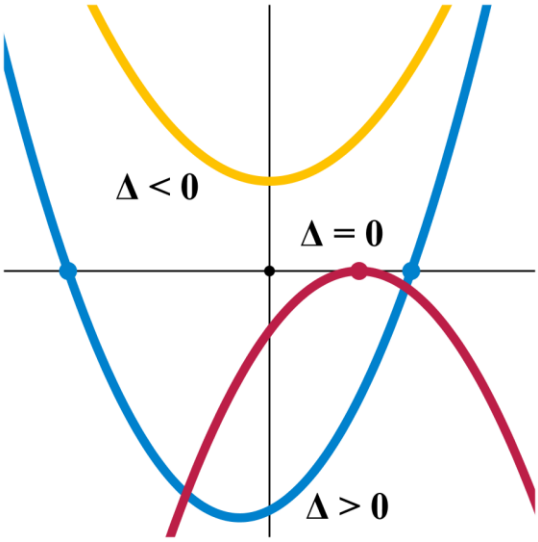
$$\frac{2-x}{3x^2} = \left(\frac{2}{3}\right)\left(\frac{-x}{x^2}\right)$$

WRONG!! Exterminate!!



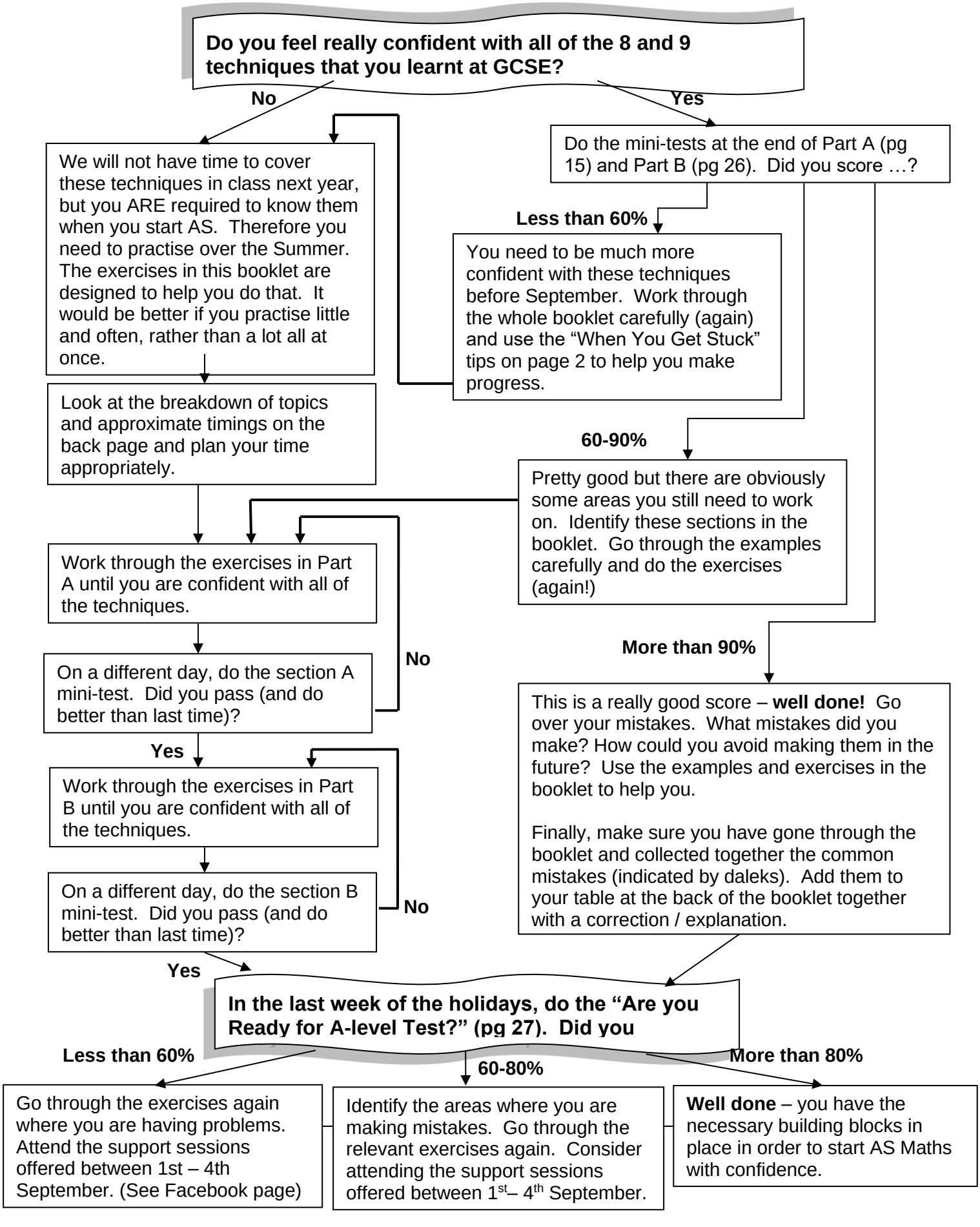
Part B – Developing Confidence with Quadratics

A quadratic is any algebraic expression with some x^2 bits and some x bits and a number i.e. $ax^2 + bx + c$. In your study of GCSE maths you will have met and learned to solve quadratic equations. In order to cope with the demands of AS Maths you need to be confident working with quadratics and this is something we have found to cause a lot of problems in the transition from GCSE to A Level Maths. This part of the booklet will outline everything you need to remember about quadratics and give you a chance to practise building your confidence with these important equations.



You should recognise these curves as quadratic curves.

Quadratic equations; trigonometry including sine, cosine rule; indices rules, simultaneous equations, …



Part A – Section 1 - FRACTIONS

TOP TIP! Never use a slanted line like this $1/2x$ because the *x* will try to escape by moving right a bit and growing

$\frac{1}{2x}$.... $1/2x \to 1/2\,x \to 1/2\;x = \frac{1}{2}x = \left(\frac{1}{2}\right)\left(\frac{x}{1}\right) = \frac{x}{2}$.

It is much harder for the *x* to escape if you use a horizontal line.

TOP TIP! You will make fewer mistakes if you write things next to each other like $3x$ rather than $3\times x$ and $\left(\frac{2}{3}\right)\left(\frac{4}{5}\right)$ rather than $\frac{2}{3}\times\frac{4}{5}$.

TOP TIP! If you want to multiply a fraction by a number, you can write the number as a fraction by putting it over 1: $5\times\frac{x}{2} = \left(\frac{5}{1}\right)\left(\frac{x}{2}\right) = \frac{5x}{2}$. This avoids the possibility of making the **common mistake** that $5\times\frac{x}{2} = \frac{5x}{10}$

Exercise 1

In the spaces available, carry out the following, leaving your answer as a single fraction.

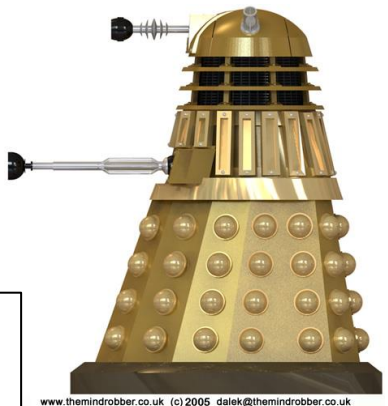
(1) $\frac{3x}{4} \times 5$ (hint: look at top tip 3 if you are not sure about this)	(2) $\frac{2}{x} + \frac{3}{x^2}$ (hint: make the denominators the same by multiplying top and bottom of $\frac{2}{x}$ by x, then add the numerators)	(3) $\frac{3x}{2} \div 5$ (hint: use top tip 3 then remember that dividing by a fraction is the same as multiplying by its reciprocal)
Answers at the back Tick when you're correct…..	Tick when correct…….	Tick when correct…….

Part A – Section 2 - INDICES

$2^{3x} = 2^3 2^x$ WRONG!!!

Students often think that if there is multiplication in the powers it must correspond to multiplication.

In fact, $2^{3x} = \left(2^x\right)^3$ or $2^{3x} = \left(2^3\right)^x = 8^x$.



$2^{x+1} = 2^x + 2^1$ WRONG!!!

Students often think that if there is addition in the power it must correspond to addition. In fact, $2^{x+1} = 2^x 2^1 = 2\left(2^x\right)$.

Exercise 2 Evaluate the following, tick the boxes when they are correct:

THE RULES OF INDICES

Rules: $a^m a^n = a^{m+n}$ $\quad \frac{a^m}{a^n} = a^{m-n}$ $\quad (a^m)^n$

Also: $(ab)^n = a^n b^n$ $\quad a^0 = 1$ $\quad a^1 = a$

Look on You Tube for The BHASVICMaths video **SwC partA sec2 INDICES 1 (ex 2)**

A negative power indicates a reciprocal

e.g 6^{-2} means $\frac{1}{6^2} = \frac{1}{36}$

and 5^{-3} means $\frac{1}{5^3} = \frac{1}{125}$

Example: $4^{\frac{3}{2}} = \left(4^{\frac{1}{2}}\right)^3 = \left(\sqrt{4}\right)^3 = 2^3 = 8$

Tick the box when you understand! ☐

A fractional power indicates a root

a power of $\frac{1}{2}$ means 'square root'. $25^{\frac{1}{2}} = \sqrt{25} = 5$

a power of $\frac{1}{3}$ means 'cube root'. $27^{\frac{1}{3}} = \sqrt[3]{27} = 3$

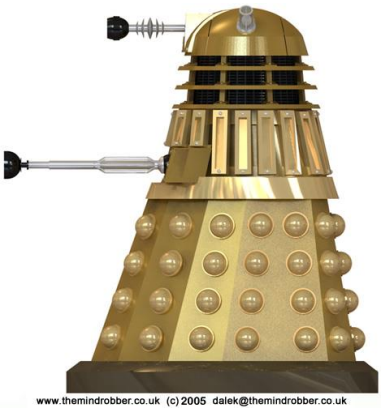
Example: $144^{-\frac{1}{2}} = \frac{1}{144^{\frac{1}{2}}} = \frac{1}{\sqrt{144}} = \frac{1}{12}$

Tick the box when you understand. ☐

(1)	2^{-6}	☐
(2)	$9^{-\frac{1}{2}}$	☐
(3)	$81^{-\frac{1}{4}}$	☐
(4)	$4^{\frac{5}{2}}$	☐
(5)	$32^{\frac{3}{5}}$	☐
(6)	$16^{-\frac{7}{4}}$	☐

Indices continued (What you need for A- level)

It is very useful to mathematicians to be able to write algebraic expressions in different ways and one of the most important ways is in the form (number) x power



$$\frac{2}{3x^2} = 6x^{-2}$$

WRONG!! Actually,

$$\frac{2}{3x^2} = \left(\frac{2}{3}\right)\left(\frac{1}{x^2}\right) = \frac{2}{3}x^{-2}$$

$$\frac{3}{x} = 3^{-x}$$

WRONG!! Actually,

$$\frac{3}{x} = \left(\frac{3}{1}\right)\left(\frac{1}{x}\right) = 3x^{-1}$$

Examples of writing things in the form αx^n. Tick the box when you understand.	**Now try Exercise 3:** Write these in the form αx^n. Tick when correct.
$\frac{2x}{3} = \left(\frac{2}{3}\right)\left(\frac{x}{1}\right)$ $= \frac{2}{3}x$ ☐	(1) $\frac{x}{5} =$ ☐
$\frac{2}{5x} = \left(\frac{2}{5}\right)\left(\frac{1}{x}\right)$ $= \frac{2}{5}x^{-1}$ ☐	(2) $\frac{3}{2\sqrt{x}} =$ ☐
$\frac{x}{3\sqrt{x}} = \left(\frac{1}{3}\right)\left(\frac{x}{\sqrt{x}}\right)$ $= \frac{1}{3}x^{1-\frac{1}{2}}$ $= \frac{1}{3}x^{\frac{1}{2}}$ ☐	(3) $\frac{\sqrt{x}}{3x^2} =$ ☐
$2\sqrt{16x^3} = 2\sqrt{16}\sqrt{x^3}$ $= 8\left(x^3\right)^{\frac{1}{2}}$ $= 8x^{\frac{3}{2}}$ ☐	(4) $\sqrt[3]{8x^2} =$ ☐
$\frac{2+x}{\sqrt{x}} = \frac{2}{\sqrt{x}} + \frac{x}{\sqrt{x}}$ $= \left(\frac{2}{1}\right)\left(\frac{1}{x^{\frac{1}{2}}}\right) + x^{\left(1-\frac{1}{2}\right)}$ $= 2x^{-\frac{1}{2}} + x^{\frac{1}{2}}$ ☐	(5) $\frac{2\sqrt{x}+4}{x^2} =$ ☐

Look on You Tube for the BHASVICMaths video **SwC partA sec2 INDICES 2 (ex 3 Q1-4)**

Look on You Tube for the BHASVICMaths video **SwC partA sec2 INDICES 3 (ex 3 Q5-14)**

Note: This one has two terms so is written in the form $\alpha x^n + \beta x^m$

More practice of **the most important type of indices…** Write these in the form $\alpha x^n + \beta x^m$. Tick the boxes when they are correct.

(6)

$$\frac{2x-4}{3x^2} = \frac{2x}{3x^2} - \frac{4}{3x^2}$$

$$= \left(\frac{2}{3}\right)\left(\frac{x}{x^2}\right) - \left(\frac{4}{3}\right)\left(\frac{1}{x^2}\right)$$

$$= \frac{2}{3}x^{-1} - \frac{4}{3}x^{-2}$$

☐

Understand?…….

(7) $\frac{1-4x}{4x^3} =$ ☐

(8) $\frac{1-4\sqrt{x}}{x} =$ ☐

(9) $\frac{x^2-3}{\sqrt{x}} =$ ☐

(10) $\frac{x-2}{x^2} =$ ☐

(11) $\frac{2+\sqrt{x}}{\sqrt{x}} =$ ☐

(12) $\frac{2x+4}{4x} =$ ☐

(13) $\frac{\sqrt{x}+6}{3x^2} =$ ☐

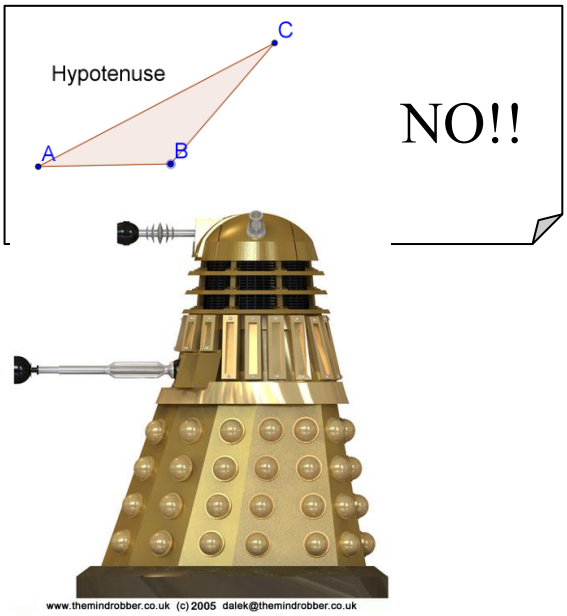
(14) $\frac{2x-1}{x^2} =$ ☐

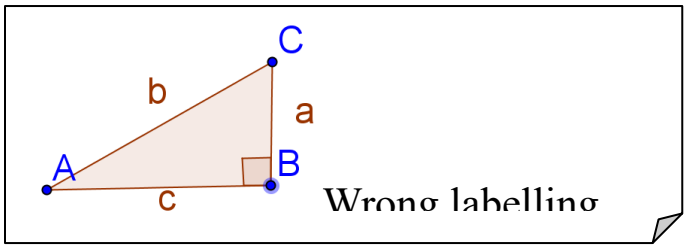
Examples of solving index equations by doing the same thing to both sides. Tick when understood.	**Exercise 3 continued:** Solve each of the following equations for *x*. Tick when correct.
$x^{-\frac{1}{2}} = 3$ $\left(x^{-\frac{1}{2}}\right)^{-1} = 3^{-1}$ $x^{\frac{1}{2}} = \frac{1}{3}$ $\left(x^{\frac{1}{2}}\right)^{2} = \left(\frac{1}{3}\right)^{2}$ $x = \frac{1^2}{3^3} = \frac{1}{9}$ ☐	(15) $x^{-\frac{2}{3}} = 9$ ☐
$x^{\frac{2}{5}} = 2$ $\left(x^{\frac{2}{5}}\right)^{5} = 2^{5}$ $x^2 = 32$ $x = \sqrt{32}$ $x = \sqrt{16}\sqrt{2}$ $x = 4\sqrt{2}$ ☐	(16) $x^{\frac{2}{5}} = 4$ ☐
$x^{\frac{2}{3}} = \frac{4}{9}$ $\left(x^{\frac{2}{3}}\right)^{\frac{1}{2}} = \left(\frac{4}{9}\right)^{\frac{1}{2}}$ $x^{\frac{1}{3}} = \frac{\sqrt{4}}{\sqrt{9}}$ $\left(x^{\frac{1}{3}}\right)^{3} = \left(\frac{2}{3}\right)^{3}$ $x = \frac{2^3}{3^3}$ $x = \frac{8}{27}$ ☐	(17) $x^{\frac{3}{4}} = \frac{1}{27}$ ☐
Note: think about how much harder this would have been if we had started by cubing both sides rather than square rooting. It would still work but it would have been more difficult.	With this question, is it easiest to start by cube rooting each side or by raising each side to the power 4?

Part A - Section 3 - TRIGONOMETRY

Trigonometry is not mainly about triangles as we will discover during the A level course. However, you do need to be able to use trigonometry to find information concerning triangles - those with a right angle and those without.

ANOTHER TOP TIP! When you see a triangle, always ask yourself two questions: does it have a right angle? How would I label it? For right angled triangles label Hypotenuse, then opposite and adjacent. For non-right angled ones, label with A, B, C on corners and a, b, c on sides

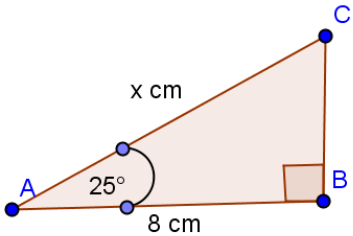
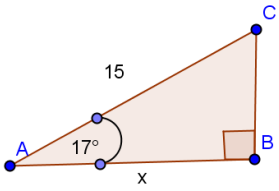
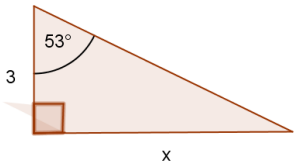


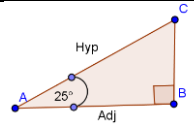


Write your method for memorising the right angle triangle rules in this box

For right angled triangles, you must learn the rules.

I use the acronym SOHCAHTOA.

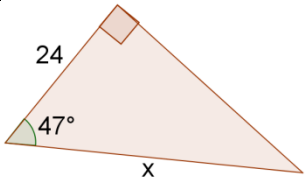
Examples. Tick when you understand.	**Now try exercise 4:** Find the side or the angle marked *x*. Tick when correct.
Find a side:  Label:	(1)  ☐
	(2)  ☐



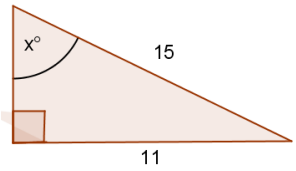
So Cos $x = \frac{A}{H}$

Cos 25 = $\frac{8}{x}$, so $x = \frac{8}{Cos\ 25}$ = 8.83 cm

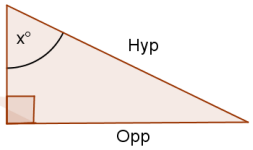
(3)



Find an angle:



Label:

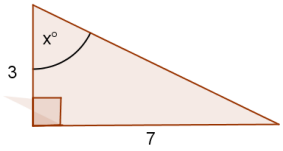


So Sin $x = \frac{O}{H}$

Sin $x = \frac{11}{15}$ = 0.7333.

x = Sin^{-1} 0.7333 = 47.2°

(4)



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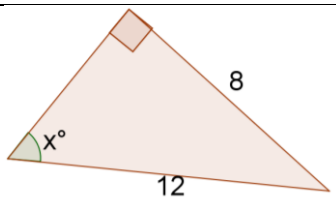


NON-RIGHTANGLE TRIANGLES

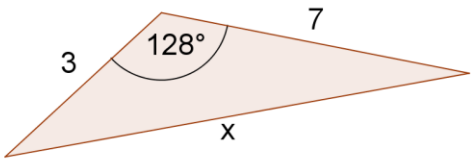
For a non-right angled triangle:

- label the corners with A, B, C.
- label the sides opposite each corner with the same letters lower case (So a, b, c).
- Choose Cosine rule if you know 2 sides and the angle between them, or if you know 3 sides
- Choose Sine rule if you know a side and an angle both having the same letter.
- Substitute and solve. and similarly for c and C

Cosine Rule: $a^2 = b^2 + c^2 - 2bcCosA$	Sin rule $\frac{a}{SinA} = \frac{b}{SinB}$

Hint: Choosing which corner to Label as A can help you so use A followed by B for angles you want to find then for angles you know.

Examples



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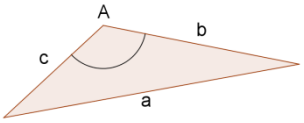
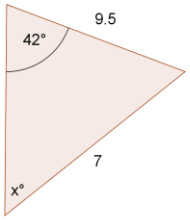


Cosine rule:

$$x^2 = 7^2 + 3^2 - 2 \times 7 \times 3 \times Cos128$$

So x^2 = 83.8578

So x = 9.16



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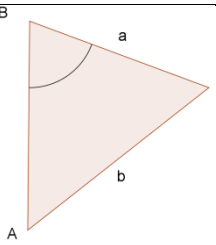


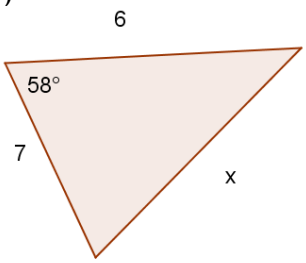
Sine rule: $\frac{9.5}{Sinx} = \frac{7}{Sin42}$

So $Sinx = \frac{sin42 \times 9.5}{7}$ = 0.9081

So x = Sin^{-1} 0.9081 = 65.2°

Exercise 4 continued. Tick when correct.

(6)



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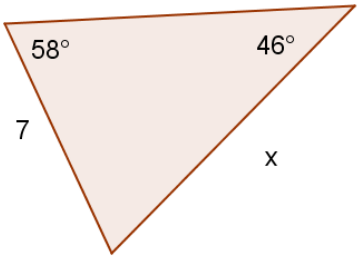

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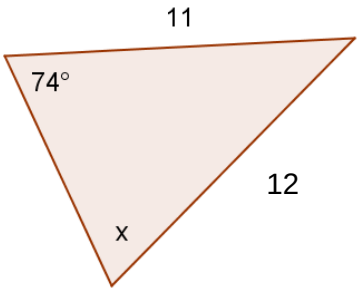

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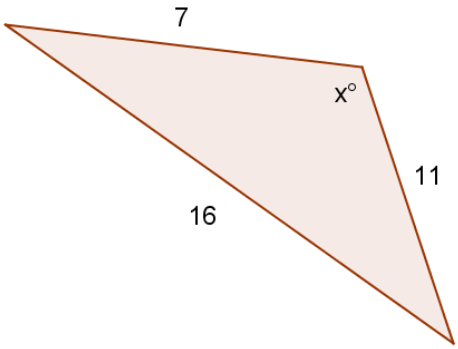


MIXED TRIGONOMETRY

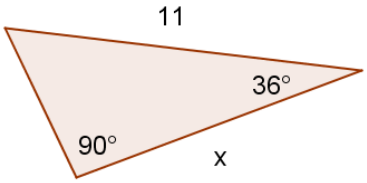
Ask yourself – right-angled triangle or not?

Reminder

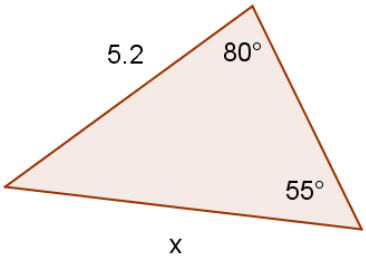
Whenever you know three sides, or you know two sides and the angle between them – use Cosine rule
Whenever you know the values of a letter, capital land lower case, - use Sine rule

Exercise 5. Find *x*. Tick when correct.

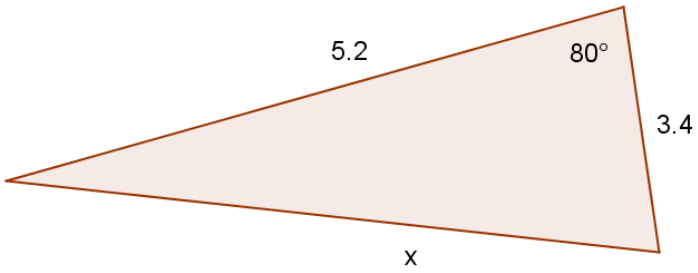
(1)



(2)



(3)



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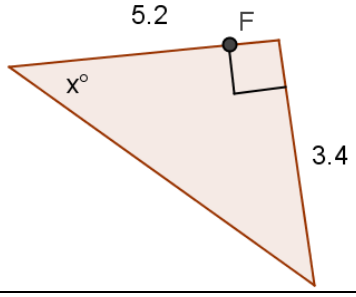


Part A Mini-Test

So, you've completed all the exercise in part A. Well done! Did you remember to copy the common mistakes you found into the table at the back of the booklet? This is really important!

The important question now is whether your brain has really learned the techniques in part A. To find out, use this mini-test in exam conditions then mark it yourself using the answers at the back of the booklet and give yourself a score. You should aim for 25/25 of course but certainly anything less than 15/25 should be a worry. Each question number comes from that number exercise. **Go back to the exercises containing the questions you got wrong** then try this test again in a few days time. If you feel you need help, follow the tips on the second page of this booklet.

Time: 45 minutes. No Calculator allowed for question 2.
Good Luck!

Cosine Rule: $a^2 = b^2 + c^2 - 2bcCosA$

Sin rule $\frac{a}{SinA} = \frac{b}{SinB}$

1 (a) Write $\frac{3x}{4} \times 5$ as a single fraction

(b) Write $\frac{2}{x} + \frac{3}{x^2}$ as a single fraction

2 (a) Evaluate $32^{\frac{3}{5}}$

(b) Evaluate $9^{-\frac{1}{2}}$

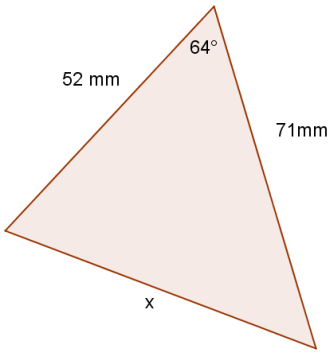
3 (a) Write $\frac{3}{2\sqrt{x}}$ in the form αx^n

(b) Write $\frac{2\sqrt{x}+4}{x^2}$ in the form $\alpha x^n + \beta x^m$

(c) Solve the equation $x^{-\frac{2}{3}} = 9$

4 Find *x*

(a)



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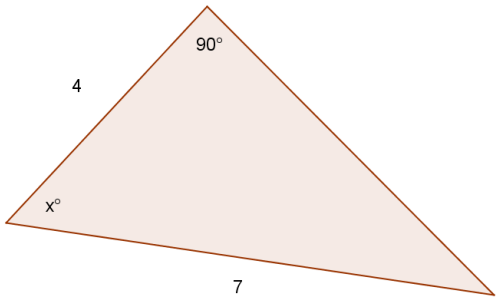

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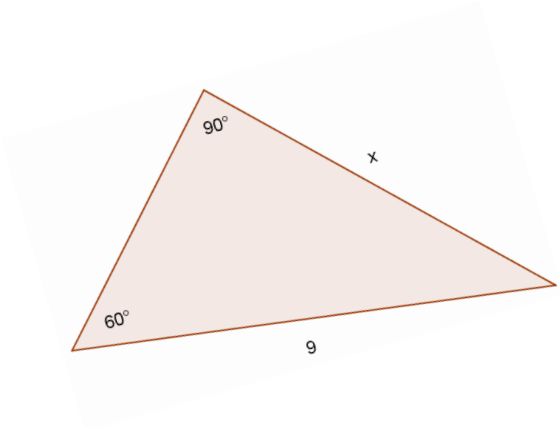

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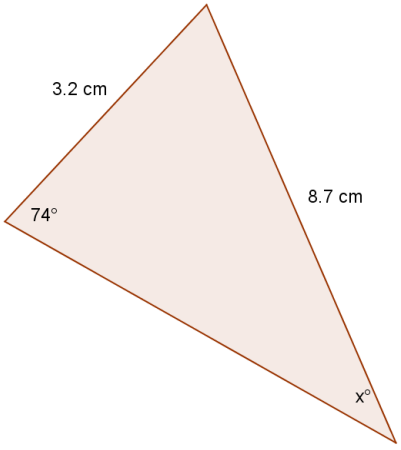


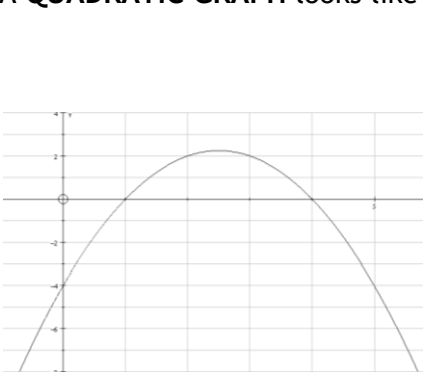
Mark your test using the solutions at the back of the booklet and put your score here /25

PART B – QUADRATICS & SIMULTANEOUS EQUATIONS

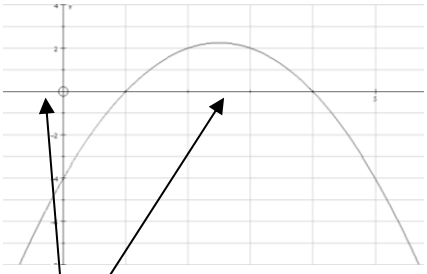
You should know what a 'quadratic' is but in order to start AS you need to REALLY understand and be able to use quadratics. You need to be able to manipulate quadratic expressions by factorising and you need to be able to solve quadratic equations by factorising or by using a formula.

A **QUADRATIC EXPRESSION** is just some algebra written in the form $ax^2 + bx + c$. The numbers a, b and c can be anything you like (b and c could even be zero!). It is usually given the name y or $f(x)$.

A **QUADRATIC GRAPH** looks like this depending on whether a is positive or negative:



A **QUADRATIC EQUATION** can always be rearranged to make the right hand side equal to zero, i.e., so that it is in the form $ax^2 + bx + c = 0$. The solutions can be <u>seen</u> (where the graph crosses the x-axis). Normally, you would expect there to be two possible answers, as in the graphs above.



Solutions to the equation $ax^2 + bx + c = 0$

Of course, if the quadratic graph is totally above or below the x axis then it will never cross the x axis. In these cases, the quadratic equation has no solutions. Or possibly the quadratic graph might just sit on the x axis rather than crossing it, in which case the quadratic equation will only have one solution (called a repeated root).

Can we solve the equation $2x^2 + 6x = 8$?!

First get everything on the left hand side so it equals zero..... $2x^2 + 6x - 8 = 0$.

You are now ready to solve the equation – if it can be solved..... This quadratic might have 2 solutions like in the picture above, it might have one solution or it might have no solutions. Over the next few pages, you will practise finding the solutions (if they exist!) by two different methods.

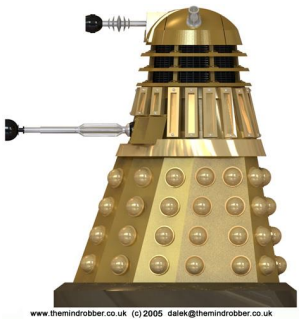
Part B - Section 1- FACTORISING QUADRATICS

First - An important concept:

Note: when squaring an expression, make sure that you square all of *it*!

If b is -6 then b^2 is $(-6)^2 = 36$ NOT -36

If b is $2k$ then $b^2 = (2k)^2 = 4k^2$ NOT $2k^2$



Second - An important skill:

… Using the difference of two squares $(a)^2 - (b)^2 = (a - b)(a + b)$

	Exercise 6 Factorise the following	Tick when correct
Example 1 $x^2 - 9 = (x - 3)(x + 3)$	(1) $x^2 - 1$	
	(2) $4x^2 - 9$	
Example 2 $9x^2 - 16 = (3x)^2 - (4)^2$ $= (3x - 4)(3x + 4)$	(3) $49 - x^2$	
	(4) $2x^2 - 8$	
	(5) $x^2 - 16$	
Example 3 $8x^2 - 2 = 2(4x^2 - 1)$ $= 2((2x)^2 - 1^2)$ $= 2(2x - 1)(2x + 1)$	(6) $9x^2 - 1$	
	(7) $36 - 25x^2$	
	(8) $9x^2 - 36$	

Exercise 7 Factorise the following quadratics. Remember to expand out to check your answers. The first one has been completed for you. Tick when correct!

(1) $x^2 - 2x - 15$	(2) $6x^2 - 3x$	(3) $x^2 - 5x - 6$
$= (x - 5)(x + 3)$ Check: $(x - 5)(x + 3)$ $= x^2 + 3x - 5x - 15$ $= x^2 - 2x - 15$ ☐	☐	☐
(4) $x^2 + x - 6$	(5) $2x^2 + 6x$	(6) $x^2 - 6x - 16$
☐	☐	☐

Exercise 8 Factorise the following. Don't forget to expand out to check your answers. Tick when correct.

(1) $2x^2 + 5x + 2$	(2) $3x^2 - 8x + 4$
☐	☐
(3) $2x^2 + 7x + 6$	(4) $3x^2 - 13x - 10$

(5) $2x^2 + 9x - 5$	(6) $2x^2 - 11x + 12$

Part B – Section 2 - SOLVING QUADRATICS

We can solve a quadratic equation by factorising or by using the quadratic formula. (There is a third way called completing the square which we will study in Term 1).

- Factorising uses the fact that if 2 things multiply together to make zero then one of them MUST be zero. You can't always factorise a quadratic even if it has solutions.
- The quadratic formula will always give you the solutions, so long as there are some!

Example - Factorising

$$2x^2 - 5x + 3 = 0$$

Factorising gives: $(2x - 3)(x - 1) = 0$

so either $2x - 3 = 0$ or $x - 1 = 0$

$2x = 3$

$\therefore x = {}^3/_2$ or $x = 1$

This means that the graph of the quadratic function $f(x) = 2x^2 - 5x + 3$ crosses the x axis at ${}^3/_2$ and 1. Tick when understood

Factorising

Exercise 9 Solve the following quadratic equations by factorising. Tick when correct.

(1) $x^2 + 11x + 28 = 0$	(2) $x^2 + 3x = 0$	(3) $2x^2 + 3x - 14 = 0$
☐	☐	☐

The Quadratic Formula

$$x = \frac{-b \pm \sqrt{(b)^2 - 4(a)(c)}}{2a}$$

Now try Exercise 10: Solve the following quadratic equations using the quadratic formula and leave your answers in the form $x = A \pm B\sqrt{C}$ as in the example on the left.

To solve the quadratic equation $ax^2 + bx + c = 0$ you can use the quadratic formula above.

Then you will need to rearrange these answers into the form

$$x = A \pm B\sqrt{C}$$

Look on You Tube for the BHASVICMaths video **SwC partB sec4 SOLVING QUADRATIC EQUATIONS 1 (ex 13)**

Example – Using the formula

Solve $x^2 + 3x + 1 = 0$

$$x = \frac{-3 \pm \sqrt{(-3)^2 - 4(1)(1)}}{2 \times 1}$$
$$= \frac{-3 \pm \sqrt{9-4}}{2}$$
$$= \frac{-3 \pm \sqrt{5}}{2}$$
$$= -\frac{3}{2} \pm \frac{\sqrt{5}}{2}$$
$$= -\frac{3}{2} \pm \left(\frac{1}{2}\right)\left(\frac{\sqrt{5}}{1}\right)$$
$$= -\frac{3}{2} \pm \frac{1}{2}\sqrt{5}$$

Important step!

Tick when understood ☐

(1) $2x^2 + 4x + 1 = 0$

(2) $x^2 - 7x + 9 = 0$

Part B – Section 3 - SOLVING SIMULTANEOUS EQUATIONS

Simultaneous equation problems involve finding the value of two (or more) unknown letters – commonly *x* and *y*. We require two equations to begin with. There are two methods with which you should be familiar.

Substitution involves finding *y* as an expression concerning *x* from one of the equations, then replacing *y* in the second equation with this expression.

e.g. solve $y + 2x = 5$

$5x - 2y = 17$

Answer. Take the first equation and rearrange it to $y = 5 - 2x$.

Now substitute this into the second equation. So $5x - 2(5 - 2x) = 17$

Solve for *x*. So $5x - 10 + 4x = 17$

$9x = 27, \qquad x = 3$

Now use $y = 5 - 2x$ to find *y*. $y = 5 - 2 \times 3 = -1$. **So *x* = 3 and *y* = -1**.

Elimination involves combining my two equations in such a way as to form a new equation which doesn't contain one of the letters. Lets look at the same problem this way.

e.g. solve $y + 2x = 5$

$5x - 2y = 17$

Answer. Take the first equation and double it to $2y + 4x = 10$.

Now write one equation under the other and add – this will eliminate *y*.

$$\begin{aligned} 2y + 4x &= 10 \\ \underline{5x - 2y} &= \underline{17} \\ 9x \qquad &= 27 \end{aligned}$$

So $x = 3$

Now use one of my original formulae (e.g. $y + 2x = 5$) to find *y*.

$y + 2 \times 3 = 5$. So $y = 5 - 6$ **So *x* = 3 and *y* = -1**.

Sometimes one method is better, sometimes the other, so make sure you understand both methods.

Exercise 11 Solve the following simultaneous equations by substitution. The first one has been started for you. Tick when correct.

(1) $y+2x=13$ $3y-5x=11\frac{1}{2}$	(2) $4x+2y=20$ $7x-y=44$	(3) $y-\frac{x}{2}=1$ $\frac{x}{3}+2y=10$
From 1st equation $y=13-2x$ So 2nd equation becomes $3(13-2x)-5x=11\frac{1}{2}$ So $39-11x=11\frac{1}{2}$ So 11*x* = 27½..		

Part B – Section 3 contined - SOLVING SIMULTANEOUS EQUATIONS using quadratics

Sometimes you will meet simultaneous equations where one equation involves a squared term (x^2 or y^2). In this case use the other equation first to find y in terms of x. **Substitution** will then give you a quadratic equation to solve. Use the methods outlined in Section A.

e.g. solve $y = x^2 - 2x$

$y - x = 4$

Answer. Take the second equation and rearrange it to $y = x + 4$.

Now substitute this into the first equation. So $x + 4 = x^2 - 2x$

Solve for x. So …

$$x^2 - 3x - 4 = 0$$

$$(x - 4)(x + 1) = 0$$

$$x = 4 \text{ or } -1$$

Notice that there are 2 possible solutions for x, so we need to find 2 solutions for y.

Use $y = x + 4$ to find y. When $x = 4$, $y = 8$ and when $x = -1$, $y = 3$

So $x = 4$ and $y = 8$ or $x = -1$ and $y = 3$

Ex 11 continued

(4) $y = 7x - 8$

$y = x^2 - x + 7$

(5) $y = x^2 - 3x + 7$

$5x + y = 15$

Part B Mini-Test

So, you've completed all the exercises in part B. Well done! The important question is whether your brain has really learned these techniques. To find out, use this mini test in exam conditions then mark it using the answers at the back of the booklet and give yourself a score. You should aim for over 80% but certainly anything less than 60% should be a worry. Go back to the exercises containing the questions you got wrong then try this test again in a few days time. If you feel you need help, follow the tips on the second page of this booklet.

Time: 30 minutes.
Good Luck!

6 Factorise the quadratic $y = 4x^2 - 9$ using the difference of two squares.

7 Factorise the quadratic $y = 2x^2 + 6x$

8 Factorise the quadratic $y = 3x^2 - 13x - 10$

9 Solve the equation $x^2 + 3x = 0$ by factorising.

10 Solve the equation $2x^2 + 4x + 1 = 0$ by using the quadratic formula, leaving the answer(s) in surd form.

11 Solve the simultaneous equations

(a)
$$3x + y = 11$$
$$9x + 2y = 28$$

(b)
$$y = 9x - 4$$
$$y = 2x^2$$

Quadratic formula: $$x = \frac{-b \pm \sqrt{(b)^2 - 4(a)(c)}}{2a}$$

Mark your test using the solutions at the back of the booklet and put your score here /20

ARE YOU READY FOR A Levels?

In order to be confident starting A Level maths you need to be confident with the techniques in this booklet. When you start the course we will give you a test like this one to check that you are ready to start A Levels. Try this test in exam conditions then mark it using the answers at the back of the booklet and give yourself a score. You should aim for over 80% but certainly anything less than 60% should be a worry. Go back to the exercises containing the questions you got wrong then try this test again in a few days time. If you feel you need help, follow the tips on the second page of this booklet.

Time: 1 hour. No Calculator allowed for question 2.
Good Luck!

Quadratic formula:

$$x = \frac{-b \pm \sqrt{(b)^2 - 4(a)(c)}}{2a}$$

Cosine Rule: $a^2 = b^2 + c^2 - 2bcCosA$

Sin rule $\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$

1(a) Write $\frac{3x}{2} \div 5$ as a single fraction

(b) Write $\frac{2}{x} + \frac{3}{x^2}$ as a single fraction

2(a) Evaluate $16^{-\frac{7}{4}}$

(b) Evaluate $4^{\frac{5}{2}}$

Staple your completed test into your booklet so that you have a record which you can discuss with your teacher in September.

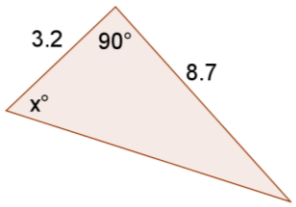
3(a) Write $\frac{2+\sqrt{x}}{\sqrt{x}}$ in the form

$\alpha x^n + \beta x^m$

(b) Solve the equation $x^{\frac{3}{4}} = \frac{1}{27}$

4 Find the value of *x* in each sketch below

(a)



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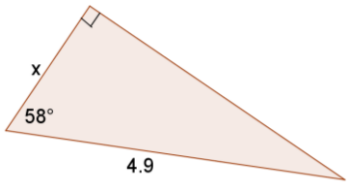

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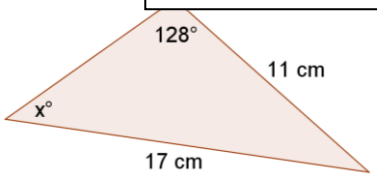
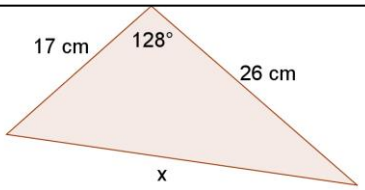




6 Factorise the quadratic $y = 2x^2 - 8$ using the difference of two squares

7 Factorise the quadratic $y = 6x^2 - 3x$

8 Factorise the quadratic $y = 2x^2 - 11x + 12$

9 Solve the equation $2x^2 + 3x - 14 = 0$ by factorising.

10 Solve the equation $x^2 - 7x + 9 = 0$ by using the quadratic formula, leaving the answer(s) in surd form.

11 Solve these simultaneous equations

(a) $4x + 2y = 20$
$7x - y = 44$

(b) $y = x^2 - 2x$
$y - x = 4$

ANSWERS

Mark your test using the solutions at the back of the booklet and put your score here /40

Exercise 1

(1) $\frac{15x}{4}$	(2) $\frac{2x+3}{x^2}$	(3) $\frac{3x}{10}$

Exercise 2

(1) $\frac{1}{64}$	(2) $\frac{1}{3}$	(3) $\frac{1}{3}$	(4) 32	(5) 8	(6) $\frac{1}{128}$

Exercise 3

(1) $\frac{1}{5}x$	(2) $\frac{3}{2}x^{-\frac{1}{2}}$	(3) $\frac{1}{3}x^{-\frac{3}{2}}$	(4) $2x^{\frac{2}{3}}$
(5) $2x^{-\frac{3}{2}}+4x^{-2}$	(6) $\frac{2}{3}x^{-1}+\frac{4}{3}x^{-2}$	(7) $\frac{1}{4}x^{-3}-x^{-2}$	(8) $x^{-1}-4x^{-\frac{1}{2}}$
(9) $x^{\frac{3}{2}}-3x^{-\frac{1}{2}}$	(10) $x^{-1}-2x^{-2}$	(11) $2x^{-\frac{1}{2}}+1$	
(12) $\frac{1}{2}+x^{-1}$			
(13) $\frac{1}{3}x^{-\frac{3}{2}}+2x^{-2}$	(14) $2x^{-1}-x^{-2}$		
(15) $x=\pm\frac{1}{27}$	(16) $x=\pm 32$	(17) $x=\frac{1}{81}$	

Exercise 4

(1) 15 Cos17 = 14.3 (2) 3Tan53 = 3.98 (3) 24/Cos47 = 35.2 (4) $\text{Tan}^{-1}(7/3)$ = 66.8°
(5) $\text{Sin}^{-1}(8/12)$ = 41.8° (6) $\sqrt{6^2+7^2-2\times 6\times 7Cos58}$ = 6.36 (7) $\frac{7sin58}{sin46}$ = 8.25
(8) $\text{sin}^{-1}(11\text{sin}74/12)$ = 61.8° (9) $\cos^{-1}(\frac{7^2+11^2-16^2}{2\times 7\times 11})$ = 123.9°

Exercise 5

(1) 11cos36 = 8.90 (2) 5.2sin80/sin55 = 6.25
(3) $\sqrt{3.4^2+5.2^2-2\times 3.4\times 5.2Cos80}$ = 5.70 (4) $\tan^{-1}\frac{3.4}{5.2}$ =33.2°

Exercise 6

(1) $(x - 1)(x + 1)$	(2) $(2x - 3)(2x + 3)$	(3) $(7 - x)(7 + x)$
(4) $2(x - 2)(x + 2)$	(5) $(x - 4)(x + 4)$	(6) $(3x - 1)(3x + 1)$
(7) $(6 - 5x)(6 + 5x)$	(8) $9(x - 2)(x + 2)$	

Exercise 7

(1) $(x + 3)(x - 5)$	(2) $3x(2x - 1)$	(3) $(x - 6)(x + 1)$
(4) $(x - 2)(x + 3)$	(5) $2x(x + 3)$	(6) $(x - 8)(x + 2)$

Exercise 8

(1) $(2x + 1)(x + 2)$	(2) $(3x - 2)(x - 2)$	(3) $(2x + 3)(x + 2)$
(4) $(3x + 2)(x - 5)$	(5) $(2x - 1)(x + 5)$	(6) $(2x - 3)(x - 4)$

Exercise 9

(1) $x = -7$ or -4	(2) $x = 0$ or -3	(3) $x = -{}^{7}/_{2}$ or 2

Exercise 10

(1) $x = -1 \pm \frac{1}{2}\sqrt{2}$	(2) $x = \frac{7}{2} \pm \frac{1}{2}\sqrt{13}$

Exercise 11

(1) $x = 2.5$, $y = 8$	(2) $x = 6$, $y = -2$	(3) $x = 6$, $y = 4$
(4) $x = 3$, $y = 13$ or $x = 5$, $y = 27$	(5) $x = 2$, $y = 5$ or $x = -4$, $y = 35$	

Part A Mini Test Solutions.

For question 1(a), give yourself 1 mark if correct. For other parts (except question 4), give yourself 2 marks for a perfect answer (including working!) , 1 mark for correct method but made a mistake and 0 marks for doing it totally wrong! Question 4: give 3 marks if perfect, 2 if method correct, 1 mark for right formula only. The total test is out of 25 and **anything below 15/25 is worrying and means you must go back to the exercises and try again to master the techniques, using the tips on page 2 of the booklet for help.**

1 (a) $\frac{3x}{4}\times 5=\left(\frac{3x}{4}\right)\left(\frac{5}{1}\right)=\frac{15x}{4}$

1 (b) $\frac{2}{x}+\frac{3}{x^2}=\frac{2x}{x^2}+\frac{3}{x^2}=\frac{2x+3}{x^2}$

2 (a) $32^{\frac{3}{5}}=\left(\sqrt[5]{32}\right)^3=2^3=8$

2 (b) $9^{-\frac{1}{2}}=\frac{1}{9^{\frac{1}{2}}}=\frac{1}{\sqrt{9}}=\frac{1}{3}$

3 (a) $\frac{3}{2\sqrt{x}}=\left(\frac{3}{2}\right)\left(\frac{1}{\sqrt{x}}\right)=\frac{3}{2}x^{-\frac{1}{2}}$

(b) $\frac{2\sqrt{x}+4}{x^2}=\frac{2\sqrt{x}}{x^2}+\frac{4}{x^2}=\left(\frac{2}{1}\right)\left(\frac{\sqrt{x}}{x^2}\right)+\left(\frac{4}{1}\right)\left(\frac{1}{x^2}\right)=2x^{\frac{1}{2}-2}+4x^{-2}=2x^{-\frac{3}{2}}+4x^{-2}$

(c)

$x^{-\frac{2}{3}}=9$

$x^{\frac{2}{3}}=\frac{1}{9}$

$x^{\frac{1}{3}}=\frac{\sqrt{1}}{\sqrt{9}}$

$x^{\frac{1}{3}}=\pm\frac{1}{3}$

$x=\left(\pm\frac{1}{3}\right)^3$

$x=\pm\frac{1}{27}$

4 (a) $c^2=52^2+71^2-2\times 52\times 71\times\cos 64$ = 4508.07, so c = 67.1

(b) $\cos x=\frac{adj}{hyp}=\frac{4}{7}=0.571$, so x = $\cos^{-1}0.571$ = 55.2°

(c) $\sin 60=\frac{opp}{hyp}=\frac{x}{9}$, so $x=9\times\sin 60=7.79$

(d) $\frac{8.7}{\sin 74} = \frac{3.2}{\sin x}$, so $\frac{\sin 74}{8.7} = \frac{\sin x}{3.2}$ so, $\sin x = \frac{3.2 \times \sin 74}{8.7} = 0.354$, so $x = \sin^{-1} 0.354$

$x = 20.7^\circ$

Part B Mini Test Solutions.

For each part except 11, give yourself 2 marks for a perfect answer (including working!), 1 mark for correct method but made a mistake and 0 marks for doing it totally wrong! Question 11: give 3 marks if perfect, 2 if only 2 answers correct, 1 mark for right initial method only. The total test is out of 16 and **anything below 10/16 is worrying and means you must go back to the exercises and try again to master the techniques, using the tips on page 2 of the booklet for help.**

6 $4x^2 - 9 = (2x-3)(2x+3)$

7 $2x^2 + 6x = 2x(x+3)$

8 $3x^2 - 13x - 10 = (3x+2)(x-5)$

9

$x^2 + 3x = 0$

$x(x+3) = 0$

$x = 0, \ x+3 = 0$

$x = 0, \ x = -3$

10

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(2)(1)}}{2(2)} = \frac{-4 \pm \sqrt{16-8}}{4} = \frac{-4 \pm \sqrt{8}}{4} = \frac{-4}{4} \pm \frac{\sqrt{8}}{4}$$

$$= -1 \pm \frac{\sqrt{4}\sqrt{2}}{4} = -1 \pm \left(\frac{2}{4}\right)\left(\frac{\sqrt{2}}{1}\right) = -1 \pm \frac{1}{2}\sqrt{2}$$

11(a)

$9x + 2y = 28$

$6x + 2y = 22$ first rule doubled

subtract

$3x = 6, \quad x = 2$

First rule gives $3 \times 2 + y = 11, \quad y = 5$

So $x = 2, y = 5$

ALTERNATIVE

$y = 11 - 3x$

$9x + 2(11 - 3x) = 28$

$3x + 22 = 28$

etc..

(b)

$9x - 4 = 2x^2$

$2x^2 - 9x + 4 = 0$

$(2x - 1)(x - 4) = 0$

$2x - 1 = 0,\ \text{so } x = 0.5,\ \text{ so } y = 9 \times 0.5 - 4 = 0.5$

$\text{OR } x - 4 = 0,\ \text{so } x = 4,\ \text{ so } y = 9 \times 4 - 4 = 32$

$\text{So } x = 0.5 \, and \ y = 0.5$

$OR \quad x = 4 \, and \ y = 32$

Are you ready for A Levels? Test Solutions.

For each part except 4 and 11, give yourself 2 marks for a perfect answer (including working!), 1 mark for correct method but made a mistake and 0 marks for doing it totally wrong! Question 4: give 3 marks if perfect, 2 if method correct, 1 mark for right formula only. Question 11: give 3 marks if perfect, 2 if only 2 out of 4 answers correct, 1 mark for right initial method only. The total test is out of 40 and **anything below 24/40 is worrying and means you must go back to the exercises and try again to master the techniques, using the tips on page 2 of the booklet for help.**

1a) $\frac{3x}{2} \div 5 = \frac{3x}{2} \times \frac{1}{5} = \frac{3x}{10}$

b) $\frac{2}{x} + \frac{3}{x^2} = \frac{2x}{x^2} + \frac{3}{x^2} = \frac{2x+3}{x^2}$

2a) $16^{-\frac{7}{4}} = \frac{1}{16^{\frac{7}{4}}} = \frac{1}{(16^{\frac{1}{4}})^7} = \frac{1}{2^7} = \frac{1}{128}$

b) $4^{\frac{5}{2}} = (4^{\frac{1}{2}})^5 = (\sqrt{4})^5 = 2^5 = 32$

3a) $\frac{2+\sqrt{x}}{\sqrt{x}} = \frac{2}{\sqrt{x}} + \frac{\sqrt{x}}{\sqrt{x}} = 2x^{-\frac{1}{2}} + 1$

b) $x^{\frac{3}{4}} = \frac{1}{27}$

$(x^{\frac{1}{4}})^3 = \frac{1}{27}$

$x^{\frac{1}{4}} = \frac{\sqrt[3]{1}}{\sqrt[3]{27}}$

$x^{\frac{1}{4}} = \frac{1}{3}$

$x = \frac{1^4}{3^4}$

$x = \frac{1}{81}$

4a) $\tan x = \frac{opp}{adj} = \frac{8.7}{3.2} = 2.719$,

so $x = \tan^{-1} 2.719 = 69.8^\circ$

b)

$\cos 58 = \frac{adj}{hyp} = \frac{x}{4.9}$, so $x = 4.9 \times \cos 58 = 2.60$

c) $\frac{17}{\sin 128} = \frac{11}{\sin x}$, so $\frac{\sin 128}{17} = \frac{\sin x}{11}$ so,

$\sin x = \frac{11 \times \sin 128}{17} = 0.510$,

so $x = \sin^{-1} 0.510 = 30.7^\circ$

d) $c^2 = 17^2 + 26^2 - 2 \times 17 \times 26 \times \cos 128$

$= 1509.2$, so $c = 38.8$

6) $2x^2 - 8 = 2(x^2 - 4) = 2(x+2)(x-2)$

7) $6x^2 - 3x = 3x(2x-1)$

8) $2x^2 - 11x + 12 = (2x-3)(x-4)$

9) $2x^2 + 3x - 14 = 0$

$(2x+7)(x-2) = 0$

$2x + 7 = 0$ or $x - 2 = 0$

$x = -\frac{7}{2}$, or $x = 2$

10) $x^2 - 7x + 9 = 0$

$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(9)}}{2(1)}$

$x = \frac{7 \pm \sqrt{13}}{2}$

$x = \frac{7}{2} \pm \frac{1}{2}\sqrt{13}$

11)

(a)

$4x + 2y = 20$

$14x - 2y = 88$ second rule doubled

add

$18x = 108, \quad x = 6$

Second rule gives $7 \times 6 - y = 44, \quad y = -2$

So $x = 6, y = -2$

(b)

$y = x + 4$

$x + 4 = x^2 - 2x$

$x^2 - 3x - 4 = 0$

$(x+1)(x-4) = 0$

$x + 1 = 0$, so $x = -1$, so $y = -1 + 4 = 3$

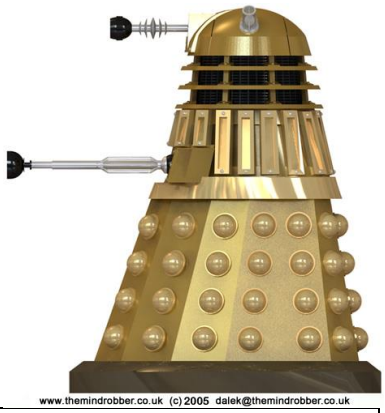
OR $x - 4 = 0$, so $x = 4$, so $y = 4 + 4 = 8$

So $x = -1 \, and \, y = 3$

OR $x = 4 \, and \, y = 8$

Notes

Use these pages to collect the top 10 most common mistakes made by students (indicated by the Daleks).Writing them down for yourself will help you not avoid doing them yourself.



Classic Mistake	Correction/Explanation

Classic Mistake	Correction/Explanation

The amount of time this booklet will take to complete will depend entirely on how well you have learned GCSE algebra and trigonometry. You will be best prepared to start A Levels if you work little and often to allow you to absorb what you are learning, then test yourself on a different day (can you still do the technique a day or a week later?) Getting started early in the holidays will give you a chance to seek help (see page 3), when (not if) you get stuck!

Use the following grid to plan and monitor your progress through the booklet.

	Topic	Exercise	Page	Approx. time required (minutes)	To be completed by	Done	Notes / Test Score
PART A	S1 - Fractions	1	6	10			
	S2 – Indices	2	7	15			
		3	8	20			
	S3 - Trigonometry	4	10	30			
		5	13	20			
	PART A Mini-Test		14	45			
PART B	S1 – Factorising Quadratics	6	17	10			
		7	18	10			
		8	18	15			
	S2 – Solving Quadratics	9	20	15			
		10	21	15			
	S3 – Solving Simultaneous equations	11	23	25			
	PART B Mini-Test		26	30			
Are You Ready for AS?			27	60			

If there is anything you would like to ask your teacher, or anything you are worried about , use this space: