Getting Ready For A Level Further Maths

Worthing College

Your Name		
A Level Further Maths		Summer 2024

Instructions: We are delighted you have chosen to study **Further Maths** at Worthing College. This pack will help you make the best possible start to studying this subject. The tasks in this pack should take you **about 4 hours to complete.** You will need to hand the work into your teacher in your ***first Further Maths lesson in September***. Make sure you keep the work safe until then, and ensure any work you hand in has your name on it. You will get feedback from your teachers on what you have done well and the challenges you will need to meet to achieve the highest possible grade. This will form part of an initial assessment of your progress your teacher will make in the first 2 weeks of the course. All of the materials you need to complete the tasks are available via the links in the document.

If you need help: The tasks are designed to get a bit more difficult as you work through them as they are preparing you for studying at a higher level and to become an effective independent learner. You should try to get as far as you can working on your own but if you do need help, please email Debbie at d.collier@worthing.ac.uk, telling her what help you need. Help is available throughout the summer holidays.

Summer work – Revision of GCSE Algebra and Introduction to Complex Numbers		
Type of task	**Task**	**Deadline**
Revision Questions	**0.1 GCSE Algebra** Complete these questions with full and careful working. Answers are provided so mark and correct your work using a different colour pen. Please bring your answers to your first further maths lesson.	First Further Maths lesson in September
Research & Questions	**0.2 Introduction to Complex Numbers** Further Maths includes work on complex numbers. This task includes some notes and questions. Read the notes very carefully and watch the videos below as necessary. Answer all the questions thoroughly. Using the answers provided, mark your work in a different colour and review anything you got wrong. Please bring your answers to your first Further Maths lesson. **Helpful videos from Khan Academy:** **Intro to the imaginary numbers** **Simplifying roots of negative numbers** **Powers of the imaginary unit** **Intro to complex numbers** **Adding complex numbers** **Subtracting complex numbers** **Multiplying complex numbers** **Solving quadratic equations: complex roots** **Dividing Complex numbers**	First Further Maths lesson in September
Challenge Questions	**0.3 Complex Numbers Challenge** A couple of exam questions on Complex numbers for you to try. You may have to do a bit more of your own research but have a go! Complete with full working and mark/annotate in a different colour. Bring to your first Further Maths lesson. **Hint**: z^* is called the **complex conjugate** of z. The real part is the same but the imaginary part has a change of sign. E.g. If $z = 3 + 4i$, then $z^* = 3 - 4i$, or if $z = -2 - 6i$ then $z^* = -2 + 6i$. When solving equations replace z with $x + iy$ and replace z^* with $x - iy$, then compare real and imaginary parts.	First Further Maths lesson in September

Work Experience week

All year 1 students are required to participate in a week-long work placement during their first year of study. You will be expected to locate one week's worth of work placement and submit your work experience form before October half term.

Placement Dates:

L2/L3 students on double /triple qualifications:	1 week course-specific placement, expected placement dates will be confirmed by the course leaders at the beginning of September.
Students with 2 or more single subjects:	1 week placement during the Easter holidays or w/c 23 June 2025

You can find the work experience form HERE
More information and guidance can be found HERE

Getting Ready For A Level Further Maths

Task 0.1 GCSE Algebra Practice

Q1.

(a) Expand and simplify $(x + 4)^2$

(b) Hence or otherwise, show that $(x + 4)^2 - 4(x + 4) \equiv x\,(x + 4)$

Q2. Simplify fully $\dfrac{18x^2 - 12x}{18x^2 - 8}$

Q3. Make x the subject of the formula

$$y = \frac{3x + 4}{x - 3}$$

Q4.

(a)
(i) Factorise $x^2 - 10x + 25$

(ii) Hence, or otherwise, solve the equation

$$(y - 3)^2 - 10(y - 3) + 25 = 0$$

(b) Simplify $\dfrac{x^2 - 9}{x^2 + 3x}$

Q5. Simplify fully $\dfrac{x^2 - 16}{3x^2 + 10x - 8}$

Q6. Solve the equation $(2x - 3)^2 = (x - 1)(x + 1)$

Give your solutions to 2 decimal places.

Q7. Solve the equation $\dfrac{x}{x + 1} - \dfrac{2}{x - 1} = 1$

Q8.

(a) Show that $(p + q)^2 \equiv p^2 + 2pq + q^2$

(b) p and q are two numbers. The sum of p and q is 10. The product of p and q is 18

Work out the value of $p^2 + q^2$

Q9.

(a) Expand and simplify $2x^2(x + 6) + 3x(x – 5)$

(b) Factorise fully $3mh^2 – 15m^2h$

(c) Simplify fully $4rs^2 \times 5r^3s^4$

(d) Solve $9x^2 + 29x – 28 = 0$

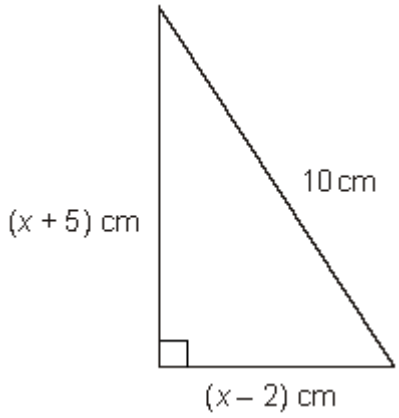
Q10.

(a)
(i) Factorise $x^2 – x – 2$

(ii) Hence, solve $x^2 – x – 2 = 0$

(b) Simplify $\dfrac{3x+2}{x^2-x-2}+\dfrac{1}{x+1}$

Q11. This right-angled triangle has sides of lengths $(x – 2)$ cm, $(x + 5)$ cm and 10 cm.



Not drawn accurately

Calculate the value of x.

Q12. Prove that $\dfrac{n}{n-4}-\dfrac{n+5}{n+6}=\dfrac{5(n+4)}{(n-4)(n+6)}$

Q13. Make x the subject of the formula $y=\dfrac{2x+3}{x-4}$

Answers

M1. $x^2 + 8x + 16$

M2. $\frac{3x}{(3x+2)}$

M3. $y(x - 3) = 3x + 4$

$yx - 3y = 3x + 4$

$yx - 3x = 3y + 4$

$x(y - 3) = 3y + 4$

$x = (3y + 4)/(y - 3)$

M4.

(a)

(i) $(x - 5)(x - 5)$ or $(x - 5)^2$

(ii) Replace x with $(y - 3)$, $y = 8$

(b) Factorise numerator: $(x - 3)(x + 3)$

Factorise denominator: $x(x + 3)$

Cancel to give $\frac{x-3}{x}$

M5. Numerator = $(x + 4)(x - 4)$

Denominator = $(3x - 2)(x + 4)$

Answer = $(x - 4)/(3x - 2)$

M6. $\frac{12 \pm \sqrt{(24)}}{6}$

2.82 **and** 1.18

M7. -1/3

M8. 64

M9.

(a) $2x^3 + 15x^2 - 15x$

(b) $3mh(h - 5m)$

(c) $20r^4s^6$

(d) $(9x - 7)(x + 4)$ 7 / 9 and –4

M10.

(a)
(i) (x + 1)(x – 2)
(ii) (x =) –1 and 2

(b) $\frac{4x}{(x+1)(x-2)}$ or $\frac{4x}{(x^2-x-2)}$

M11. $(x + 5)^2 + (x - 2)^2 = 10^2$ Answer to 1dp = 4.6,

M12. $\frac{n(n+6)}{(n-4)(n+6)}(-)\frac{(n+5)(n-4)}{(n-4)(n+6)}$

M13. $y(x - 4) = 2x + 3$

$xy - 4y = 2x + 3$

$xy - 2x = 4y + 3$

$x(y - 2) = 4y + 3$

(x =) $\frac{(4y+3)}{(y-2)}$

Getting Ready For A Level Further Maths
Task 0.2 – Introduction to Complex Numbers

The square root of a negative number

Until now you may have learnt that you cannot square root a negative number, but now you are going to see how this is possible in the following way:
Suppose $i^2 = -1$ then we can deduce that $\sqrt{-1} = i$
This means we can use i every time we want to find the square root of a negative number.

Example: $\sqrt{-9} = \sqrt{9}\sqrt{-1} = 3i$ Check this by working backwards: $(3i)^2 = 9i^2 = 9 \times -1 = -9$

Powers of i

We can find other powers of i also:

Examples: $i^3 = i^2 \times i = -1 \times i = -i$ and $i^4 = i^2 \times i^2 = -1 \times -1 = 1$ etc….

Calculations with Complex Numbers

Complex numbers are any numbers that have i in them, such as $3 + 4i$ or $2 - 7i$.
You can calculate with them as long as you remember that $i^2 = -1$.

Examples: $(3 + 4i) + (5 - 7i) = 8 - 3i$
$(2 - 3i)(3 + 5i) = 6 - 9i + 10i - 15i^2 = 6 + i - (15 \times -1) = 6 + i + 15 = 21 + i$

***Remember to watch the videos for fuller explanations and demonstrations.**

Exercise

Now try these examples, simplifying as far as possible:

1) a) $\sqrt{-25}$ b) $\sqrt{-49}$ c) $\sqrt{-121}$

2) a) i^3 b) i^4 c) i^5 d) i^8 e) i^{34} f) i^{-1}

3) a) $(4 - 7i) + (2 - 6i)$ b) $(3 + 8i) - (1 + 5i)$ c) $(11 + i) - (12 - 2i)$

4) a) $(2 + 5i)(3 + 2i)$ b) $(3 + 7i)(4 - i)$ c) $(6 - 4i)(7 - 8i)$

5) Use the quadratic formula to solve for x and give answers as simplified as possible:

a) $x^2 + 4x + 5 = 0$ b) $x^2 + 6x + 11 = 0$ c) $2x^2 + 3x + 2 = 0$

6) Try these divisions, making the denominator real in the same way you would rationalise the denominator using surds (remember that $i = \sqrt{-1}$, so it is just like working with surds!)

a) $\frac{3+4i}{2+i}$ b) $\frac{22-2i}{4-5i}$ c) $\frac{2+4i}{5-3i}$

Answers

1) a) 5i b) 7i c) 11i

2) a) -i b) 1 c) i d) 1 e) -1 f) -i

3) a) 6 – 13i b) 2 + 3i c) -1 + 3i

4) a) -4 + 19i b) 19 + 25i c) 10 – 76i

5) a) -2 ± i b) $-3 \pm \sqrt{2}i$ c) $\frac{-3}{4} \pm \frac{\sqrt{7}}{4}i$

6) a) 2 + i b) $\frac{98}{41} + \frac{102}{41}i$ c) $\frac{-1}{17} + \frac{13}{17}i$

Getting Ready For A Level Further Maths
Task 0.3 – Complex Numbers Challenge – Exam Questions
Answers are provided but make sure you show a full method

Q1.

(a) Solve the equation $w^2 + 6w + 34 = 0$, giving your answers in the form $p + q\text{i}$, where p and q are integers.

(3)

(b) It is given that $z = \text{i}(1 + \text{i})(2 + \text{i})$.

(i) Express z in the form $a + b\text{i}$, where a and b are integers.

(3)

(ii) Find integers m and n such that $z + mz^* = n\text{i}$.

(3)
(Total 9 marks)

Q2.

(a) Solve the following equations, giving each root in the form $a + b\text{i}$:

(i) $x^2 + 9 = 0$;

(1)

(ii) $(x + 2)^2 + 9 = 0$.

(1)

(b) (i) Expand $(1 + x)^3$.

(1)

(ii) Express $(1 + 2\text{i})^3$ in the form $a + b\text{i}$.

(3)

(iii) Given that $z = 1 + 2\text{i}$, find the value of

$$z^* - z^3$$

(2)
(Total 8 marks)

Answers

M1.(a)

$= -3 \pm 5i$

(b) (i)

$= -3 + i$

(ii)

$\Rightarrow m = -1, n = 2$

M2.(a) (i) $x = \pm 3i$

(ii) $x = -2 \pm 3i$

(b) (i) $(1 + x)^3 = 1 + 3x + 3x^2 + x^3$

(ii)$= -11 - 2i$

(iii)

$= 12$